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A note to users of Ritchey-Chretien telescopes looking to get the most from their instruments.

In this paper we focus on answering the question "how precise does my collimation and mirror spacing need to be in order to obtain all of the imaging performance that my observing site and other equipment are capable of delivering?"

Preface

Case 1; Long exposure imaging:

We begin by calculating the angular extent of the Airy disc and noting for all cases under current consideration, that the Airy disc diameter is smaller than one arc second (see table 1 below). If we accept a priori that the best atmospheric "seeing" conditions at our site will probably not support resolution **in a long-exposure image** better than 1 arcsecond, (an optimistic estimate for many of us), then a reasonable error budget target for image degradations introduced by collimation and mirror spacing errors might be to require that the **RMS change in the geometric RMS spot size**[1] at the center of the field not be allowed to exceed a value of 0.5 arcsecond. The decision to adopt an error budget target derived from the geometric spot size is justified by the fact that under long exposure conditions, atmospheric turbulence is assumed to dominate the intensity distribution in the stellar image.

Adopting a value for the RMS change in RMS spot size of 0.5 arcsecond for our Long-exposure error budget insures that residual aberrations in the telescope will not enlarge the angular diameter of stellar images formed under a 1 arcsesond seeing-imposed limit by more than about 12%.

It should also be pointed out that when mounted on the RC telescopes considered in this study, the angular sizes of two commonly available CCD pixels (see table 1) will limit the resolution of extended source detail to angular values that equal twice the angular extent of the pixel (the Nyquist sampling limit). Hence, for all cases herein considered, the error budget we propose will not appreciably degrade telescope performance below the limit already imposed by the finite size of the CCD detector elements.

Table 1 Angular diameters of the Airy disc compared against the angular sizes of two common CCD pixels. Results presented for four commercially available different RC telescope designs.

Design	Angular diameter of Airy disc	Angular extent of 9 micron CCD pixel	Angular extent of 25 micron CCD pixel
12.5" F/9 Ritchey-Chretien	0.87 arc-sec	0.65 arc-sec	1.81 arc-sec
14.5" F/7.9 Ritchey-Chretien	0.75 arc-sec	0.64 arc-sec	1.77 arc-sec
16" F/8.4 Ritchey-Chretien	0.68 arc-sec	0.54 arc-sec	1.51 arc-sec
20" F/8.1 Ritchey-Chretien	0.55 arc-sec	0.45 arc-sec	1.25 arc-sec

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[1] The RMS change, ΔRMS, of a quantity or function Z(ρ,θ) as used in this paper is defined by: $(\Delta RMS)^2 = \frac{1}{p}\int_0^{2p}\int_0^1 (Z' - Z)^2\, r\,dr\,dq$, in which ρ, θ are understood to be the polar coordinates of the unit circle representing either transverse displacements in the image plane (RMS spot size), or in the exit pupil (RMS wavefront error); and Z' represents the quantity in question degraded by the presence of residual collimation errors (spot size or wavefront error).

Case 2; Visual observation & short exposure imaging:

If, however, we are concerned with predicting the impact that alignment errors will have on either visual observation of low contrast detail, or on **short exposure imaging** (i.e. video frame grabbing, sorting, and subsequent stacking), then the situation is complicated by the fact that during some finite small percentage of the detection intervals, the seeing will allow resolution at or near the diffraction limit. For short exposure conditions, diffraction effects on the intensity distribution at the image plane cannot be ignored, and a more useful (and stringent) error budget metric would only allow collimation errors to exist at a magnitude that preserved the as-aligned modulation transfer function (MTF) above a chosen value at spatial frequencies of interest to the observer.

A general requirement for a useful tolerance sensitivity analysis is to choose a figure of merit by which the impact of tolerance degradations on the performance of the nominal system can be described as a single number. Since different observers will, in general, be interested in resolving different spatial frequencies, calculating the impact on the MTF at a single, arbitrarily chosen spatial frequency will not yield the figure of merit needed to enable such an analysis to move forward.

Instead, we propose the idea that changes in the Strehl ratio can be used to track the impact of residual aberrations near the diffraction limit. The Strehl ratio is a single number that is proportional to the integrated area under the MTF surface, and thus encompasses the impact that aberrations have on the MTF at all spatial frequencies below the diffraction limited cutoff frequency.

Noting that a Strehl value of 0.80 is a commonly accepted definition for diffraction-limited imaging performance, one possible approach for establishing a short exposure error budget target would be to require that the sum of residual aberrations introduced by alignment errors and the errors left in the telescope by the optical fabrication process should not degrade the Strehl ratio for an axial object below a chosen value, say of 0.75.

Recognizing in advance that there will not be universal agreement on the best value for the degraded Strehl ratio to adopt as an error budget target, we justify our choice of 0.75 by observing that the manufacturer's of commercially available RC optics typically do not guaranty wavefront performance **as measured in their laboratory** to be any better than that needed to support a Strehl ratio value of 0.80; hence, our proposed limit of 0.75 allows for a small degradation below the limit imposed by the optical fabrication process itself.

To put the analysis in terms of a quantity that can be measured in the optical fabrication facility, the 0.75 limit on the Strehl ratio can be recast in terms of an allowable change in RMS wavefront error. A useful approximation for the Strehl ratio (accurate for Strehl ratios > 0.1) is given by: $S.R. \cong 1 - (2\pi\sigma)^2$, in which σ represent the RMS wavefront error calculated at the system exit pupil. Most commercial vendors of RC telescopes nowadays guarantee optics (when properly collimated and spaced) that will support an RMS wavefront error of $\leq 0.072\lambda$ RMS. By rearranging the expression given for S.R. above to solve for σ, we determine that a S.R. = 0.75 corresponds to a total allowed RMS WFE (wavefront error) $\sigma_T = 0.080$. Thus the allowable degradation (or RMS change) in the RMS WFE, σ_C, due to collimation errors is found to be; $\boldsymbol{\sigma_C = \mathrm{SQRT}(0.080^2 - 0.072^2) = 0.035\lambda}$ **RMS**.[2]

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[2] An additional advantage of the decision to choose the RMS change in RMS wavefront error as the tolerance budget metric is that for systems near the diffraction limit, small changes in the RMS change are linear with respect to small changes in the magnitude of the tolerance perturbations. Hence, once a sensitivity analysis has been performed, the system error budget can be re-calculated for any arbitrarily chosen set of tolerance value inputs, so long as the magnitude of the degradations introduced do not cause the RMS change to grow beyond a value of approximately 0.10 – 0.12 waves RMS.

Note also that the error budget as defined above makes no allowance for the degradation in the off-axis imagery caused by the residual field curvature and astigmatism inherent to the RC optical design. Observers wishing to achieve the widest possible flat field performance are urged to review the first two papers in this series for suggestions on how the off-axis performance of RC telescopes may be improved by the addition of corrector lens assemblies to the telescope.

Type of alignment errors considered.

The alignment errors considered in this analysis can be categorized and described as follows:

1. Error in setting the vertex separation between the primary and secondary mirrors introduces an image displacement along the telescope axis, and also introduces spherical aberration. The image shift can be compensated by re-focusing. The impact of spherical aberration cannot be removed by focusing; however the overall WFE can be minimized by the choice of focus position.

2. Centration errors between the primary and secondary mirrors, with respect to the mechanical axis established by the focuser mechanics. Four possible scenarios are investigated:

    a. The focuser is accurately centered with respect to (WRT) to the primary mirror, M1; the secondary mirror, M2 is de-centered.
    b. Both M1 & M2 are centered WRT each other, but the focuser is de-centered relative to M1.
    c. Both the focuser and M2 are de-centered WRT M1, but in the same direction.
    d. Both the focuser and M2 are de-centered WRT M1, but in opposite directions.

Throughout it is assumed that the user of the telescope is familiar with and has used the star test to remove coma from a star image at the center of the field. Thus the tip and tilt adjustments provided at M1 & M2 have been applied **before** the RMS change values are calculated.

1. Effect of mirror spacing error.

The primary effects of mirror spacing errors are described as an image displacement along the optical axis and the introduction of spherical aberration everywhere in the field. The magnitude of the image shift may be readily calculated as the product of the spacing error and the square of the secondary mirror amplification ratio. For most practical R-C systems, therefore, the image shift is somewhat less than 9x the mirror spacing error, and if the spacing error is less than, say, 0.25 inches, the resulting image shift may (usually) be accommodated within the range of travel available at the focuser.

More problematic is the introduction of spherical aberration, because the presence of this aberration rapidly degrades the contrast transfer across the entire field, as the magnitude of the aberration approaches and then exceeds the diffraction-limited value of 0.072 λ RMS. At the diffraction-limit, the presence of fourth-order spherical aberration reduces the value of the MTF by approximately 25-30% in the mid-range spatial frequencies. (For an F/9 system used in the visible spectral range, the mid-range will correspond to frequencies in the range 30 - 120 cycles / mm.) Since the Nyquist frequency of many commonly used CCD detectors falls somewhere in the range 25 – 60 cycles/mm, the introduction of spherical aberration at a magnitude $\approx 0.072\,\lambda$ RMS should be treated as a serious problem, to be avoided if at all possible.

Fortunately, the above-mentioned image shift can serve as a warning sign that something is amiss before the error becomes large enough to introduce 0.072λ RMS of spherical aberration.

To estimate the magnitude of the mirror spacing error tolerable for **short exposure** conditions, Table 2 below summarizes the amount of mirror spacing error that introduces 0.035 λ RMS spherical aberration into the four designs investigated in this monograph. Note that the aberration introduced by other amounts of spacing error scale linearly with the ratio of the spacing errors.

Table 2 Mirror spacing errors introducing 0.035 λ RMS spherical aberration @ λ = 550 nm.

Tolerable spacing error (inches)	**Design**			
	12.5” F/9	**14.5” F/7.9**	**16” F/8.4**	**20” F/8.1**
	0.074”	0.041”	0.065”	0.056”

The effect of spherical aberration on **long exposure images** is tabulated in table 3, where the spacing error that causes the RMS change in the RMS spot size to reach a value of 0.5 arc-second.

Table 3 Mirror spacing errors causing a 0.5 arc-sec RMS change in RMS spot size @ @ λ = 550 nm

Tolerable spacing error (inches)	**Design**			
	12.5” F/9	**14.5” F/7.9**	**16” F/8.4**	**20” F/8.1**
	0.220”	0.150”	0.274”	0.285”

Conclusion:

Comparing tables 2 and 3, we see that the tolerance on spacing error needed to support long-exposure imaging at the (somewhat arbitrarily chosen) seeing limit of 1 arc-second is approximately 3.0X less stringent than the tolerance required to achieve near diffraction-limited visual or short exposure performance. However, verifying and adjusting (if necessary) the airspace to within either tolerance set should be an achievable task, using the star test at 30-50X per inch of aperture on a steady night to detect residual spherical aberration. Another observation is that the allowable tolerance is a strong function of primary mirror F/#, becoming more stringent as the F/# decreases. That this is so is not surprising, since the spherical aberration present in the airspace between M1 and M2 varies as the fourth power (OPD units of measure) of the F/#, and de-spacing M1 and M2 disturbs the extent to which the undercorrection introduced by M2 balances the overcorrected aberration introduced at M1.

Effect of misalignment between optical and mechanical axes:

Comprehensive analysis of the effect that uncompensated mirror & focuser centration errors have on image quality in RC telescopes is complicated by the fact that the analysis must allow for the possibility that the mechanical axis defined by the focuser may not be aligned (centration & tilt) with respect to either of the two mirrors in the system. Furthermore, my analysis using ZEMAX has determined that the impact that centering errors have depends on the extent to which the resulting coma has been compensated by adjusting the tip & tilt controls provided for both M1 (primary mirror) & M2 (secondary mirror). Useful results, therefore, can only be obtained by communicating clearly the assumptions that were applied in making the calculations. Three important assumptions made in the present analysis are:

1. The mechanical axis of the focuser is initially fixed, and serves as a reference datum for the entire telescope system.

2. The angular orientations of both mirrors (M1 & M2 tip & tilt) are applied as free variables to remove coma from a star image located at the center of the field. The center field position is at the intersection of the optical focal plane and the focuser mechanical axis.

3. Any residual mis-match between the angular orientation of the focuser axis and the image plane of best definition has been compensated **after** alignment of M1 & M2 has occurred to optimize the on-axis image quality. Such compensation may be implemented in practice by examination of a wide-field image, followed by differential shimming of the focuser.

The analysis uses a 16” F/8.4 Ritchey-Chretien design as an example.

Table 4 Sensitivity analysis for 16” F/8.4 RC telescope: Impact of secondary mirror & focuser de-center errors on RMS wavefront error at the center and extents of a 1” diameter image circle.

Scenario[1]	**De-center (inches)**[2]	**RMS WFE @ 550 nm**				
		Center	**Top**	**Bottom**	**Left**	**Right**
Nominal Design	0.0	0.0016	0.158	0.158	0.158	0.158
1	0.08	0.173	0.080	0.499	0.278	0.278
2	0.08	0.0026	0.211	0.109	0.159	0.159
3	0.08	0.127	0.133	0.432	0.251	0.251
4	0.08	0.224	0.022	0.570	0.314	0.314

Table 5 Sensitivity analysis for 16” F/8.4 RC telescope: Impact of secondary mirror & focuser de-center errors on RMS change in RMS wavefront error at the center and extents of a 1” diameter image circle.

Scenario[1]	**De-center (inches)**[2]	**RMS change in WFE @ 550 nm**				
		Center	**Top**	**Bottom**	**Left**	**Right**
Nominal Design	0.0	0.000	0.000	0.000	0.000	0.000
1	0.08	0.173	0.136	0.473	0.229	0.229
2	0.08	0.002	0.140	0.114	0.018	0.018
3	0.08	0.127	0.085	0.402	0.195	0.195
4	0.08	0.224	0.133	0.370	0.114	0.114

[1]
1. Focuser centered on primary mirror. Secondary mirror de-centered.
2. Secondary mirror centered, focuser de-centered with respect to primary mirror.
3. Both focuser and secondary mirror are de-centered , but in same direction.
4. Both focuser and secondary are de-centered, but in opposite directions.

[2] Both de-center values are evaluated at the same linear magnitude. Direction of de-center is always aligned parallel to the off-axis directions identified in the tables as “top” and “bottom

Table 6 Allowable centration errors, satisfying the constraint: RMS change in WFE =0.035 l RMS at the center of the field.

Scenario	**Max tolerable decenter (inches)**	**RMS spot size on-axis (arcsec)**	**RMS change in WFE on-axis**	**Worst case RMS change 0.5” off-axis**
1	0.016	0.83	0.035	0.096
2	0.025	0.68	0.000	0.035
3	0.022	0.68	0.035	0.111
4	0.013	1.08	0.035	0.058

Discussion

Tables 4 and 5 contain the results used to generate table 6. In table 4, the actual values of the RMS wavefront error are shown for the four alignment scenarios investigated (see footnotes to table 5 for scenario summary). These values were calculated after the perturbations for each scenario were introduced, and the telescope model re-optimized to remove coma from the on-axis image. Note that the actual location of the on-axis object in object space was allowed to float during the optimization; the only constraint was that the star image appearing in the center of the focuser be coma-free). Next, the RMS change in the RMS wavefront error is calculated, and these values displayed in table 5. Finally, the values for maximum allowable de-center under each scenario are calculated by scaling the 0.080" de-center used to calculate the sensitivities (table 4 & 5) by the ratio 0.035/actual RMS change. Also shown in table 6 are values for the RMS spot diameter, in units of arc-seconds.

Several important observations regarding the sensitivity of RC scopes to mirror & focuser de-centers can be made:

- The allowable de-center is always between 0.01 – 0.03 inches, which is 4 -10x more stringent than the calculated tolerance for the M1-M2 vertex separation.
- After M1 & M2 are adjusted to remove coma, the image quality over the extended format is no longer symmetrical about the center of the field. This is true even though we have allowed for differential shimming of the focuser to remove image plane tilt, because the source of the residual asymmetry is a variation of astigmatism with field position that cannot be entirely compensated by re-focusing.
- With the exception of scenario #2 (M1 & M2 centered, focuser de-centered), residual astigmatism also exists at the center field point; hence the expected result is that the star test of the optimized system with de-centers present will show an astigmatic appearance, of a magnitude that is proportional to the de-center(s) present in the system.
- The worst-case asymmetry over the extended image format occurs when both M2 & the focuser are de-centered in the same direction (scenario 3). The worst-case astigmatic residual at the center of field position occurs when M2 & the focuser are de-centered in opposite directions (scenario 4).

Because astigmatism at the center of the field can be introduced by causes other than the presence of de-centers (mirror figure errors, mirrors bending in their cells), **unambiguous evidence for the presence of residual de-centers requires an evaluation of image quality over an extended field, checking for the symmetry of the aberrated star images about the field center point.**